INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Consulting point for the development of cooperation in the area operation innovation and transfer technology

The activity:

A COLLABORATION ON THE HIGH PRECISION MACHINING METHODS

Theoretical Theoretical and applicable technological bases II

Learning texts

Doc. Ing. Jaroslav Prokop, CSc.
FSI VUT v Brně

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Contents:

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Introduction

The object of this project is the analysis and specification of issues to be solved in the real application in the field. Each area is focused on the technological characteristics and requirements of the relevant technological systems. As far as possible are processed parts with specific examples for better orientation of potential users.

1 Accuracy and quality of machined surfaces

Accuracy and quality of the machined surface is an integrated output of the manufacturing process.
Parameters assessed the accuracy and quality of machined surface as specifying precision parameters which include:

size deviation - deviation from the nominal value
shape deviation - deviation from straightness deviation of circularity, cylindricity deviation
position deviation - deviation from parallelism, perpendicularity deviation, deviation alignment
surface structure - the arithmetic average deviation Ra, the greatest height of the profile Rz

In some special cases can be quantified other parameters such as type and size of stress in the surface layer of the machined surface, the microhardness of the surface layer

Parameters specified accuracy and surface quality depends on many technological factors that may be considered by their nature divided into:

- systematically constant - error in setting machine, deviation size and shape of the tool
- systematically variable - tool wear, thermal deformation of machine
- random - allowances for machining distraction, distraction of material properties

Parameters precision and surface quality are quantified for the identified machining process, especially when it identifies workpiece machining method, machine tool, tool and cutting conditions.

Precision machined surfaces is generally a function of precision and technological properties of machine, tool, workpiece, fixture and cutting conditions.

Machine to machine for accuracy machined surfaces usually given priority status and its properties generally exert a decisive influence realized precision parameters machined surface.

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2. Quality control of machined surface in production process

- Quality control and measurements.
- Measuring of dimensions.
- Measuring of shapes.
- Position errors.
- Surface roughness assesment.

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3. Statistical interpretation of parameters machined surface accuracy

The precision of a machined surface is quantified according to the machining of a number of carefully chosen test pieces depending on the technological aspects of the identified machining process.
To generalize the results of the analysis it is important to identify the conditions under which the quantified parameters of the machined surface precision were investigated. For practical reasons it is important to identify machine method, machine tool, test work-piece, tool and cutting conditions. The parameters of accuracy are specified for the identified machining process and for the evaluated test piece surface and the methodology of their measurement is designed. Fundamental characteristics of the measurement instruments are also the part of the measuring procedures.
Deviations of the machined surface have generally the character of continuous random variables and their values are quantified on the base of machining a number of test work-pieces. The number of test work-pieces is generally denoted as n and it is chosen with respect to the expected progression and trends of assessed deviation and the character of the machining processes. For the stable machining processes, in which technology influences on the accuracy are largely accidental nature, it is possible to recommend $n \geq 5$. For the case, where the trend of the change in the accuracy parameters is evident and where systematically variable influences prevail, it will be necessary to choose larger number of test pieces.
Statistical interpretation of the precision parameters of the assigned machined surface is based on the assumption of the progression and trends of the assessed values. The formulation of these assumptions or hypotheses results from knowledge of similar or analogous machining processes. Methodology and findings of the final analysis will be used depending on the input assumptions and hypotheses. There are machining processes which correspond to a particular statistical distribution of evaluated parameters and machining processes in which the distribution of assessed values is unknown. In the analysis of machining process parameters in terms of their accuracy it is often used a normal distribution, the hypothesis of normal distribution of random variable can be verified by an appropriate test of normality.

3.1 Normal distribution of the precision parameter of machined surface

Normal distribution of the precision parameter of machined surface is particularly applicable in those cases where the random nature of the technological factors prevails and where systematical variable influences are corrected or eliminated during the machining process. These conditions meet requirements for the machining process carried out on CNC machine tool with a diagnosis of tool condition and thermal deformations or for the machining process carried out on general-purpose machine tool with qualified staff in the short-run production.
Input data for the statistical interpretation are the precision parameters of the machined surface carried out on n work-pieces, which are generally labeled $x_1, x_2 \dots x_i \dots x_n$.
These values are considered to be random sample from a normally distributed population, which is characterized by mean **m** and standard deviation **σ**.
Methodology differentiates depending on whether or not we know parameters of normal distribution of the assessed precision parameters of machined surface. Usually neither mean **m** nor standard deviation **σ** is not known and therefore we use assessments. For selected

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parameters of precision of the machined surface it is quantified the estimated mean, confidence interval of mean and statistical tolerance interval.

3.1.1 The estimated mean of the precision parameter of machined surface

The estimated mean of the precision parameter of machined surface is marked and is qualified as sample mean defined by:

$$\bar{x} = \frac{1}{n} \sum_{i} x_i \quad (1)$$

3.1.2 Confidence interval of the mean of the precision parameter of machined surface

The estimated mean of the precision parameter of machined surface x is random value itself. In terms of this fact one determines two-sided or one-sided confidence interval of the mean of the precision parameter of machined surface. Limits of the confidence interval restrict the mean of the precision parameter of machined surface with certain confidence level.

Two-sided confidence interval of the mean of the precision parameter of machined surface is limited by limits given by:

$$P\,(\,m_{D2} < m < m_{H2}\,) = 1 - \alpha \quad (2)$$

m_{D2} - lower limit of the two-sided confidence interval of the mean of the precision parameter of machined surface

m_{H2} - upper limit of the two-sided confidence interval of the mean of the precision parameter of machined surface

m - mean of the precision parameter of machine d surface

$1 - \alpha$ - confidence level

$$P\,(\,m_{D1} < m\,) = 1 - \alpha \quad (3)$$

$$P\,(\,m < m_{H1}\,) = 1 - \alpha \quad (4)$$

m_{D1} - lower limit of the one-sided confidence interval of the mean of the precision parameter of machined surface

m_{H1} - upper limit of the one-sided confidence interval of the mean of the precision parameter of machined surface

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Confidence intervals limits of the mean of the precision parameter of machined surface are calculated on the basis of the estimated mean $\bar{x}$ and estimated standard deviation s according to the following relations:

$$m_{D2} = \bar{x} - t_{1-\alpha/2;\, n-1} \frac{s}{\sqrt{n}} \qquad (5)$$

$$m_{H2} = \bar{x} + t_{1-\alpha/2;\, n-1} \frac{s}{\sqrt{n}} \qquad (6)$$

$$m_{D1} = \bar{x} - t_{1-\alpha;\, n-1} \frac{s}{\sqrt{n}} \qquad (7)$$

$$m_{H1} = \bar{x} + t_{1-\alpha;\, n-1} \frac{s}{\sqrt{n}} \qquad (8)$$

$t_{1-\alpha/2;n-1}$ - **1-∝/2** –quantile of Student's t-distribution with **(n-1)** degrees of freedom

$t_{1-\alpha;n-1}$ - **1-∝** - quantile of Student's t-distribution with **(n-1)** degrees of freedom

s - estimated standard deviation of the precision parameter of machined surface

Quantiles of Student's t-distribution values are in [2], [3], [5]. Selected values of q - quantile of Student's t-distribution for **ν** degrees of freedom are shown in Appendix 3.1.
Estimated standard deviation of the precision parameter of machined surface is given by the relation:

$$s = \sqrt{\frac{1}{n-1} \sum_i (x_i - \bar{x})^2} \qquad (9)$$

The size of two-sided confidence interval of the mean of the precision parameter of machined surface is denoted $\mathbf{I_{m2}}$ and is calculated as a difference between appropriate limits:

$$I_{m2} = m_{H2} - m_{D2} = 2t_{1-\alpha/2;n-1} \frac{s}{\sqrt{n}} \qquad (10)$$

3.1.3 Statistical tolerance interval of the precision parameter of machined surface

Statistical tolerance interval of the precision parameter of machined surface is the interval, for which there is fixed confidence level **1-α** indicating that it will cover the part **p** of the population, from which the random sample comes from.

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Statistical tolerance interval is set as the two-sided or one-sided, their limits are given by:

$$L_{i2} = \bar{x} - k_2 \cdot s \qquad (11)$$

$$L_{s2} = \bar{x} + k_2 \cdot s \qquad (12)$$

$$L_{i1} = \bar{x} - k_1 \cdot s \qquad (13)$$

$$L_{s1} = \bar{x} + k_1 \cdot s \qquad (14)$$

L_{i2} - lower limit of the two-sided statistical tolerance interval of the precision parameter of machined surface

L_{s2} - upper limit of the two-sided statistical tolerance interval of the precision parameter of machined surface

L_{i1} - lower limit of the one-sided statistical tolerance interval of the precision parameter of machined surface

L_{s1} - upper limit of the one-sided statistical tolerance interval of the precision parameter of machined surface

k_2 - coefficient for limits of the two-sided statistical tolerance interval of the precision parameter of machined surface

k_1 - coefficient for limits of the one-sided statistical tolerance interval of the precision parameter of machined surface

The values of coefficient $\mathbf{k_2}$, $\mathbf{k_1}$ depends on the number of examined test work-pieces **n**, on the chosen proportion of the population **p** to be covered by given limits and on the chosen confidence level **1 - α**. The values of coefficient $\mathbf{k_2}$ **(n, p, 1-α)** and $\mathbf{k_1}$ **(n, p, 1-α)** are for example in [2], [4]. Selected values of coefficient $\mathbf{k_2}$ a $\mathbf{k_1}$ for normal distribution of the examined value with unknown values **m** a **σ** are mentioned in appendix 3.2 a 3.3.

The size of the statistical tolerance interval of the precision parameter of machined surface $\mathbf{I_2}$ is given as a difference between appropriate upper and lower limit:

$$I_2 = L_{s2} - L_{i2} = 2\, k_2 \cdot s \qquad (15)$$

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3.2 Unknown distribution of the precision parameter of machined surface

In case of unknown, but continuous distribution of the assessed values it is possible to use one of the distribution-free method for the statistical interpretation of the precision of the tested machined surface.

The statistical interpretation refers to extremes of the examined specified precision parameters in terms of this procedure.

The estimated mean of the precision parameter $\bar{x}$ and the estimated standard deviation of the precision parameter **s** is determined on the basis of the precision parameters of machined surface identified on the tested pieces $x_1, x_2 \ldots x_i \ldots x_n$. Values $\bar{x}$ and s are calculated according to relations (3.1) and (3.9) shown above. These estimations are informative for next proceeding.

Statistical interpretation of the precision parameter is carried out in relation to the minimal and maximal value of the examined precision parameters x_{min} , x_{max}, given by:

$$x_{min} = \min \{x_1, x_2 \ldots x_i \ldots x_n\}$$

$$x_{max} = \max \{x_1, x_2 \ldots x_i \ldots x_n\}$$

In the term of the formal procedure we have one-sided spread and the two-sided spread of the assessed values related to one-sided and two-sided statistical tolerance interval.

3.1.1 One-sided spread of the precision parameter

One-sided spread of the precision parameter is based on the assumption that between the number of test pieces **n**, confidence level **1-α** and the proportion **p** superior to x_{min} or inferior to x_{max} one has the relation:

$$p^n = \alpha \qquad (16)$$

The solution is done on the basis of the analysis of this relation, when one has two given or chosen values and the third is specified. In general three following basic cases can happen.

a) Probability **(1–α)** that the proportion of the population **p** is superior to x_{min} (or inferior to x_{max})

$$(1-\alpha) = 1 - p^n \qquad (17)$$

b) Proportion of the population **p**, which is with the probability **(1 – α)** superior to x_{min}

(or inferior to x_{max})

a) Probability **(1–α)** that the proportion of the population **p** is superior to $\mathbf{x_{min}}$ (or inferior to $\mathbf{x_{max}}$)

$$(1-\alpha) = 1 - p^{n} \tag{17}$$

b) Proportion of the population **p**, which is with the probability **(1 – α)** superior to $\mathbf{x_{min}}$ (or inferior to $\mathbf{x_{max}}$)

$$p = \left[1 - (1-\alpha)\right]^{\frac{1}{n}} \tag{18}$$

c) Number of test pieces **n**, when the proportion of the population **p** is with the probability **(1 – α)** in the interval

$$n = \frac{\log\left[1 - (1-\alpha)\right]}{\log p} \tag{19}$$

3.1.1 Two-sided spread of the precision parameter

Two-sided spread of the precision parameter is based on the assumption that between the number of test pieces **n**, confidence level **1-α** and the proportion **p** which is between $\mathbf{x_{min}}$ and $\mathbf{x_{max}}$ one has the relation:

$$n \,.\, p^{n-1} - (n-1) \,.\, p^{n} = \alpha \tag{20}$$

In general the solution of the problem is done for three following cases:

a) Probability **(1–α)** that the proportion of the population **p** lies in the interval $< \mathbf{x_{min}}, \mathbf{x_{max}} >$

$$(1-\alpha) = 1 - n \,.\, p^{n-1} + (n-1) \,.\, p^{n} \tag{21}$$

b) Proportion of the population **p**, which is with the probability **(1 – α)** in the interval $< \mathbf{x_{min}}, \mathbf{x_{max}} >$

The size of proportion of the population **p** is determined by the step-by-step solution of (3.22).

$$(1-\alpha) - 1 + n \,.\, p^{n-1} = (n-1) \,.\, p^{n} \tag{22}$$

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c) The number of test pieces when the proportion of the population **p** is with the probability

(1 – α) in the interval $< \mathbf{x_{min}},\ \mathbf{x_{max}} >$

The value **n** is determined by the step-by-step solution of (22).

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Chosen values of q-quantile Student T distribution for ν degees of freedom t_q

	q				
	0,90	0,95	0,975	0,99	0,995
4	1,533	2,132	2,776	3,747	4,604
5	1,476	2,015	2,571	3,365	4,032
6	1,440	1,943	2,447	3,143	3,707
7	1,415	1,895	2,365	2,998	3,499
8	1,397	1,860	2,306	2,896	3,355
9	1,383	1,833	2,262	2,821	3,250
10	1,372	1,812	2,228	2,764	3,169
11	1,363	1,796	2,201	2,718	3,106
12	1,356	1,782	2,179	2,681	3,055
13	1,350	1,771	2,160	2,650	3,012
14	1,345	1,761	2,145	2,624	2,977
15	1,341	1,753	2,131	2,602	2,947
16	1,337	1,746	2,120	2,583	2,921
17	1,333	1,740	2,110	2,567	2,898
18	1,330	1,734	2,101	2,552	2,878
19	1,328	1,729	2,093	2,539	2,861
20	1,325	1,725	2,086	2,528	2,845
21	1,323	1,721	2,080	2,518	2,831
22	1,321	1,717	2,074	2,508	2,819
23	1,319	1,714	2,069	2,500	2,807
24	1,318	1,711	2,064	2,492	2,797
25	1,316	1,708	2,060	2,485	2,787
26	1,315	1,706	2,056	2,479	2,779
27	1,314	1,703	2,052	2,473	2,771
28	1,313	1,701	2,048	2,467	2,763
29	1,311	1,699	2,045	2,462	2,756
30	1,310	1,697	2,042	2,457	2,750

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Chosen values coefficient k_2 (n, p, 1-α) to determine Two-sided spread of the precision parameter - normal distribution - m and σ unknown

n	1-α = 0,95			1-α = 0,99		
	p = 0,90	p = 0,95	p = 0,99	p = 0,90	p = 0,95	p = 0,99
5	4,28	5,08	6,63	6,61	7,86	10,26
6	3,71	4,41	5,78	5,34	6,35	8,30
7	3,37	4,01	5,25	4,61	5,49	7,19
8	3,14	3,73	4,89	4,15	4,94	6,47
9	2,97	3,53	4,63	3,82	4,55	5,97
10	2,84	3,38	4,43	3,58	4,27	5,59
11	2,74	3,26	4,28	3,40	4,05	5,31
12	2,66	3,16	4,15	3,25	3,87	5,08
13	2,59	3,08	4,04	3,13	3,13	4,89
14	2,53	3,01	3,96	3,03	3,61	4,74
15	2,48	2,95	3,88	2,95	3,51	4,61
16	2,44	2,90	3,81	2,87	3,41	4,49
17	2,40	2,86	3,75	2,81	3,35	4,39
18	2,37	2,82	3,70	2,72	3,28	4,31
19	2,34	2,78	3,66	2,70	3,22	4,23
20	2,31	2,75	3,62	2,66	3,17	4,16
21	2,29	2,72	3,58	2,62	3,12	4,10
22	2,26	2,70	3,54	2,58	3,08	4,04
23	2,24	2,67	3,51	2,56	3,04	3,99
24	2,23	2,65	3,48	2,52	3,00	3,96
25	2,21	2,63	3,46	2,49	2,97	3,90
26	2,19	2,61	3,43	2,47	2,94	3,87
27	2,18	2,59	3,41	2,45	2,91	3,83
28	2,16	2,58	3,39	2,43	2,89	3,79
29	2,15	2,56	3,37	2,40	2,86	3,76
30	2,14	2,55	3,35	2,39	2,84	3,73

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Chosen values coefficient k_2 (n, p, 1-α) to determine one-sided spread of the precision parameter - normal distribution - m and σ unknown

n	1-α = 0,95			1-α = 0,99		
	p = 0,90	p = 0,95	p = 0,99	p = 0,90	p = 0,95	p = 0,99
5	3,41	4,21	5,75			
6	3,01	3,71	5,07	4,41	5,41	7,33
7	2,76	3,40	4,64	3,86	4,73	6,41
8	2,58	3,19	4,36	3,50	4,29	5,81
9	2,45	3,03	4,14	3,24	3,97	5,39
10	2,36	2,91	3,98	3,05	3,74	5,08
11	2,28	2,82	3,85	2,90	3,56	4,83
12	2,21	2,74	3,75	2,77	3,41	4,63
13	2,16	2,67	3,66	2,68	3,29	4,47
14	2,11	2,61	3,59	2,59	3,19	4,34
15	2,07	2,57	3,52	2,52	3,10	4,22
16	2,03	2,52	3,46	2,46	3,03	4,12
17	2,00	2,49	3,41	2,41	2,96	4,04
18	1,97	2,45	3,37	2,36	2,91	3,96
19	1,95	2,42	3,33	2,32	2,86	3,89
20	1,93	2,40	3,30	2,28	2,81	3,83
21	1,91	2,37	3,26	2,24	2,77	3,78
22	1,89	2,35	3,23	2,21	2,73	3,73
23	1,87	2,33	3,21	2,18	2,69	3,68
24	1,85	2,31	3,18	2,15	2,66	3,64
25	1,84	2,29	3,16	2,13	2,63	3,60
26	1,82	2,27	3,13	2,11	2,60	3,56
27	1,81	2,26	3,12	2,09	2,58	3,53
28	1,80	2,24	3,09	2,07	2,56	3,50
29	1,79	2,23	3,08	2,05	2,54	3,47
30	1,78	2,22	3,06	2,03	2,52	3,45

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1-α	p					
	0,50	0,75	0,90	0,95	0,99	0,999
0,50	1	3	7	14	60	693
0,75	3	5	14	28	138	1386
0,90	4	9	22	45	230	2302
0,95	5	11	29	59	299	2995
0,99	7	17	44	90	459	4603
0,999	10	25	66	135	688	6905

1-α	p					
	0,50	0,75	0,90	0,95	0,99	0,999
0,50	3	7	17	34	168	1679
0,75	5	10	27	53	269	2692
0,90	7	15	38	77	388	3889
0,95	8	18	46	93	473	4742
0,99	11	24	64	130	662	6636
0,999	14	33	89	181	920	9230

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